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# Effect of laser radiation upon heat and mass transfer in two-component elastic semitransparent layer

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### Abstract

In present paper the effect of the correlation between spectral radiative characteristics of different lasers and absorptive characteristics of laser irradiated two-component elastic semitransparent material upon coupled thermal, diffusive and elastic processes in the layer is examined. Irradiated material is supposed to consist of elastic matrix and gaseous admixture. Investigations are carried out within the model applied early to the study of mentioned coupled processes in the layer subjected to thermal infrared radiation. Calculations were carried out for four different infrared lasers. Peculiarities of heat and admixture mass transfer caused by laser irradiation are established and discussed. © 2003 Elsevier Ltd. All rights reserved.

## 1. Introduction

Various aspects of the effect of laser radiation upon solid materials are extensively studied in the literature, both theoretically and experimentally. Majority of theoretical studies are focused on examination of a thermal effect of laser radiation upon material. A relatively little attention was paid to investigation of the correlation between spectral radiative characteristics of the laser and absorptive characteristics of irradiated material, namely, the correlations between laser radiation bands and absoption bands of a material irradiated by a laser. Since in most technical applications homogeneous one-component metallic materials, absorbing in wide range of a spectrum due to presence of free electrons, are subjected to laser radiation the consideration of mentioned correlation is not so important. This correlation becomes important, however, when semitransparent materials are subjected to laser radiation because they absorb radiation selectively. The effect is even more pronounced if multicomponent solid is irradiated because of specific response of different components to radiation.

Effect of selective heating of certain components of multicomponent solid caused by components' different response to radiation due to their different absorbing characteristics was reported, for example, in papers [1-3]. Possibility of fast volumetric selective heating by electromagnetic radiation of the condensed phase in heterogeneous systems is indicated in [4]. Obtaining of composite materials with improved mechanical properties in experimental works [2,3] has been ascribed to different response of different composite's constituents to external radiation resulting in their different diffusion rate. Therefore, technological problems related to radiation action upon multicomponent solid claims for construction of a model to capture coupled mechanical, thermal and diffusive processes in such solids subjected to radiation. For the case of thermal infrared radiation appropriate model was constructed recently [5-7]. The present paper is a continuation of these studies by expansion for the case of nonthermal laser radiation characterized, unlike thermal radiation, by discrete band structure.

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# Nomenclature

а	absorption coefficient, $m^{-1}$		
$Bi_T$	Biot criterium for heat exchange		
$Bi_c$	Biot criterium for mass exchange		
с	dimensionless admixture concentration		
$\bar{c}$	velocity of radiation propagation in vac-		
	uum. m/s		
d	admixture molecule diameter, m		
D	diffusivity. m <sup>2</sup> /s		
$D_0$	diffusion factor. $m^2/s$		
Ē	molar energy. J/mol		
$E_{\rm act}$	diffusion activation energy, J/mol		
$\overline{E}$	Young modulus, Pa		
F	ponderomotive force, N/m <sup>3</sup>		
h	thickness of the layer, m		
$h_m$	thickness of boundary monolayer, m		
I	radiation intensity, $W/(m^2 \text{ ster } \mu m)$		
J	intensity of absorption band [11]. $(cm^2 Pa)^{-1}$		
J	mass flux, $kg/(m^2 s)$		
$k_T$	thermal diffusivity. $m^2/s$		
$L_4$	parameter defined by Eq. (15)		
m	mass of monolayer, kg		
М	molar weight, kg/mol		
n	refraction index		
N, S	number of absorptive and radiative bands		
	respectively		
$N_{\rm pt}$	number of photons corresponding to inten-		
F.	sity of radiation, $(m^2 s ster)^{-1}$		
р	number of diffusive jumps, s <sup>-1</sup>		
$q_A$	mass transfer heat, J/kg		
$\hat{Q}$	power of heat source per unit mass, W/kg		
R	universal gas constant, J/(K mol)		
$R_{\lambda}$	reflectivity		
t	time, s		
Т	temperature, K		
w	coefficient of linear concentration expansion		
	(see Eq. (18))		
$Z_*$	thickness coordinate, m		
Ζ	dimensionless coordinate, $z = z_*/h$		
Creak a	wahala		
Greek S	$V^{(n)}$		
$\alpha_T$	(see Eq.(18))		
~	(SUC Eq. (10))		
α	boundary		
	boundary		

β	direction of laser beam propagation in the		
	layer		
$\gamma_A$	coefficient of adhesion, $(m^2/s)^{-1}$ (Eq. (21))		
$\Delta \lambda_n$	width of radiation band, $\mu m$		
$\Delta \lambda_{n_A}^{er}$	width of absorption band, µm		
$\Delta_{\lambda_n \lambda_{n_A}}$	width of bands' intersection, µm		
$\theta_A$	mass exchange rate at layer's boundary, kg/ $(m^2 s)$		
$\theta_{2}$	optical thickness at wavelength $\lambda_n$		
$\kappa$	thermal conductivity, W/(mK)		
λ	wavelength, µm		
$\overline{v}$	Poisson ratio		
ξ	direction of laser beam incidence		
ρ	density, kg/m <sup>3</sup>		
σ	stress tensor, Pa		
$ au^A$	molecule's lifetime in absorbed state, s		
$ au_c$	dimensionless time in Eq. (12)		
$ au_T$	dimensionless time in Eq. (11)		
$\psi_A$	energy flux to admixture, W/kg		
Subscripts			
Subscrip	ots		
Subscrip A, M	<i>ts</i> quantities related to admixture and matrix		
Subscrip A, M	ots quantities related to admixture and matrix respectively		
Subscrip Α, Μ λ	ots quantities related to admixture and matrix respectively spectral quantity		
Subscrip A, M λ 0	ots quantities related to admixture and matrix respectively spectral quantity initial value		
Subscrip A, M λ 0 (p)	<i>ats</i> quantities related to admixture and matrix respectively spectral quantity initial value number of iteration		
Subscrip A, M $\lambda$ 0 (p) s	ots quantities related to admixture and matrix respectively spectral quantity initial value number of iteration quantity related to surface		
Subscrip A, M $\lambda$ 0 (p) s sur	ots quantities related to admixture and matrix respectively spectral quantity initial value number of iteration quantity related to surface quantity related to surroundings		
Subscrip A, M $\lambda$ 0 (p) s sur x, y, z	quantities related to admixture and matrix respectively spectral quantity initial value number of iteration quantity related to surface quantity related to surroundings indices defining components of pondero-		
Subscrip A, M $\lambda$ 0 (p) s sur x, y, z	quantities related to admixture and matrix respectively spectral quantity initial value number of iteration quantity related to surface quantity related to surroundings indices defining components of pondero- motive forces and stress tensor		
Subscrip A, M $\lambda$ 0 (p) s sur x, y, z Superscr	<i>ets</i> quantities related to admixture and matrix respectively spectral quantity initial value number of iteration quantity related to surface quantity related to surroundings indices defining components of pondero- motive forces and stress tensor		
Subscrip A, M $\lambda$ 0 (p) s sur x, y, z Superscrip	<i>quantities</i> related to admixture and matrix respectively spectral quantity initial value number of iteration quantity related to surface quantity related to surroundings indices defining components of pondero- motive forces and stress tensor <i>ripts</i> incident		
Subscrip A, M $\lambda$ 0 (p) s sur x, y, z Superscription S	<i>ts</i> quantities related to admixture and matrix respectively spectral quantity initial value number of iteration quantity related to surface quantity related to surroundings indices defining components of pondero- motive forces and stress tensor <i>tipts</i> incident source of radiation		
Subscrip A, M $\lambda$ 0 (p) s sur x, y, z Superscription S atm	<i>vis</i> quantities related to admixture and matrix respectively spectral quantity initial value number of iteration quantity related to surface quantity related to surroundings indices defining components of pondero- motive forces and stress tensor <i>ripts</i> incident source of radiation value calculated at atmospheric pressure		
Subscrip A, M $\lambda$ 0 (p) s sur x, y, z Superscription S atm	quantities related to admixture and matrix respectively spectral quantity initial value number of iteration quantity related to surface quantity related to surroundings indices defining components of pondero- motive forces and stress tensor <i>ripts</i> incident source of radiation value calculated at atmospheric pressure according to Clapeyron equation		
Subscrip A, M $\lambda$ 0 (p) s sur x, y, z Superscription S atm A, M	<i>vis</i> quantities related to admixture and matrix respectively spectral quantity initial value number of iteration quantity related to surface quantity related to surroundings indices defining components of pondero- motive forces and stress tensor <i>vipts</i> incident source of radiation value calculated at atmospheric pressure according to Clapeyron equation quantities related to admixture and matrix		
Subscrip A, M $\lambda$ 0 (p) s sur x, y, z Superscrip S atm A, M	<i>vis</i> quantities related to admixture and matrix respectively spectral quantity initial value number of iteration quantity related to surface quantity related to surroundings indices defining components of pondero- motive forces and stress tensor <i>vipts</i> incident source of radiation value calculated at atmospheric pressure according to Clapeyron equation quantities related to admixture and matrix respectively		
Subscrip A, M $\lambda$ 0 (p) s sur x, y, z Superscription S atm A, M sur	<i>vis</i> quantities related to admixture and matrix respectively spectral quantity initial value number of iteration quantity related to surface quantity related to surroundings indices defining components of pondero- motive forces and stress tensor <i>vipts</i> incident source of radiation value calculated at atmospheric pressure according to Clapeyron equation quantities related to admixture and matrix respectively quantity related to surroundings		

### *n* number of band

- *A* quantity related to admixture
- *z* coordinate direction

The aim of the present study is to examine the effect of correlation between laser spectral radiation bands and admixture absorption bands upon thermal and diffusive processes in elastic semitransparent two-component solid layer, consisting of solid matrix and admixture, subjected to laser irradiation.

# 2. Radiation equations

Consider an infinite semitransparent two-component layer of thickness *h* consisting of elastic matrix and gaseous admixture. Layer's surroundings  $z_* < 0$  are vacuumed while domain  $z_* > h$  represents real gaseous medium of a given admixture concentration  $c_{sur}$ . Suppose layer's surface  $z_* = 0$  is uniformly irradiated by collimated laser beam creating angle  $\xi_0$  with positive direction of axis  $z_*$  (Fig. 1). Spectral intensity of incident radiation in *n*th band of laser radiation is given by expression

$$I_{\lambda_n}^{\mathbf{i}} = k I_{\lambda_n}^{\mathbf{S}} \delta(\xi - \xi_0), \quad 0 \leqslant \xi \leqslant \pi/2.$$
(1)

Here  $I_{\lambda_n}^{S}$  is spectral intensity of laser radiation source and  $\delta(\xi)$  is Dirac's delta-function, *k* is proportionality coefficient. We shall consider laser radiation of moderate spectral intensity that satisfies condition [8]

$$I_{\lambda_n}^{\mathbf{S}} < \frac{(n_{\lambda_n})^2}{k\alpha(1 - R_{\lambda_n}(\xi_0))}, \quad n = \overline{1, S}.$$
 (2)

Above constraint being observed, the laser radiation propagation in considered semitransparent layer is described by the linear radiation transfer equation which within two-flux model in an approximation of a cold nonscattering medium takes the form [9]

$$\frac{\mathrm{d}I_{\lambda_n}^{\pm}(z,\nu)}{\mathrm{d}z}\nu \pm ha_{\lambda_n}I_{\lambda_n}^{\pm}(z,\nu) = 0. \tag{3}$$

 $I_{\lambda_n}^{\pm}(z,v)$  here are spectral intensities of radiation propagating in the layer in positive and negative directions of z-axis respectively,  $v = \cos \beta$ , and

$$a_{\lambda_n} = a_{\lambda_n}(c_A) = a_{\lambda_n}^M + \sum_{n_A=1}^N a_{\lambda_n}^{n_A} c_A$$
(4)

is absorption coefficient of the layer in *n*th band of laser radiation while  $a_{\lambda_n}^M$  and  $\sum_{n_A=1}^N a_{\lambda_n}^{n_A} c_A$  are appropriate coefficients of matrix and admixture. Here

$$a_{\lambda_n}^{n_A} = a_{\lambda_n_A}^* \Delta_{\lambda_n \lambda_{n_A}} / \Delta \lambda_{n_A}^{\text{ef}}$$
<sup>(5)</sup>

is absorption coefficient of admixture at its density correspondent to atmospheric pressure from Clapey-ron's equation;  $a_{\lambda_{n_A}}^* = \rho_M / (\rho_A^{\text{atm}} \Delta \lambda_{n_A}^{\text{ef}})$ .



Fig. 1. The schematics of the system.

Solutions of Eq. (3) are

$$I_{\lambda_{n}}^{+}(z,v) = I_{\lambda_{n}}^{+}(0,v) \exp\left\{-\int_{0}^{z} \frac{\theta_{\lambda_{n}}(c_{A})}{v} dz^{*}\right\},\$$
$$I_{\lambda_{n}}^{-}(z,v) = I_{\lambda_{n}}^{-}(1,v) \exp\left\{\int_{1}^{z} \frac{\theta_{\lambda_{n}}(c_{A})}{v} dz^{*}\right\}.$$
(6)

Assuming layer's surfaces z = 0, 1 to reflect diffusively with reflectivity  $R_{\lambda_n}$  while refractive index be equal to its averaged over heating and admixture concentration change interval value,  $n_{\lambda_n}$ , and taking into account Eq. (1), we arrive at following system of integral equations for effective boundary intensities

$$\begin{split} I_{\lambda_{n}}^{+}(0,v) &- 2R_{\lambda_{n}} \int_{0}^{1} \left\{ vI_{\lambda_{n}}^{-}(1,v) \exp\left(-\int_{0}^{1} \frac{\theta_{\lambda_{n}}(c_{A})}{v} dz\right) \right\} dv \\ &= \left\{ \begin{pmatrix} n_{\lambda_{n}}(1-R_{\lambda_{n}})kI_{\lambda_{n}}^{S}\delta(\xi-\xi_{0}) & \text{if } v \leq v_{*}, \\ 0 & \text{if } v > v_{*}, \end{pmatrix} \\ I_{\lambda_{n}}^{-}(1,v) &- 2R_{\lambda_{n}} \int_{0}^{1} \left\{ vI_{\lambda_{n}}^{+}(0,v) \exp\left(-\int_{0}^{1} \frac{\theta_{\lambda_{n}}(c_{A})}{v} dz\right) \right\} dv = 0, \end{split}$$

$$(7)$$

where  $\xi = \sin^{-1}(n_{\lambda_n}\sqrt{1-v^2})$ ,  $v_* = \cos\beta_*$ ,  $\beta_* = \sin^{-1}(1/n_{\lambda_n})$ . Eqs. (3)–(7) form the set of equations to determine spectral radiation intensity in the layer.

# 3. Effect of laser radiation upon the layer: Model equations

Our aim is to study coupled thermal and diffusive processes in elastic semitransparent solid with gaseous admixture subjected to laser (nonthermal) radiation. With this purpose we shall use the model [5,6] previously developed by the authors that describes above processes in semitransparent solid subjected to thermal radiation. According to the model, effect of external radiation upon considered solid is expressed in terms of heat sources  $\rho_M Q$ , ponderomotive force  $F_z$ , energy flux to admixture  $\psi_A$ , all those being functions of spectral intensities of radiation  $I^+_{\lambda}$ ,  $I^-_{\lambda}$ . Taking into account expression (4) for absorption coefficient, following expressions for mentioned quantities are obtained

$$\rho_{M}Q = 2\pi \sum_{n=1}^{S} \left( a_{\lambda_{n}}^{M} \Delta \lambda_{n} + \sum_{n_{A}=1}^{N} a_{\lambda_{n}}^{*} \varDelta_{\lambda_{n}\lambda_{n_{A}}} c_{A} \right) \\
\times \int_{0}^{1} \left( I_{\lambda_{n}}^{+}(z, v) + I_{\lambda_{n}}^{-}(z, v) \right) \mathrm{d}v;$$
(8)

$$F_{z} = 2\pi \bar{c}^{-1} \sum_{n=1}^{S} \left( a_{\lambda_{n}}^{M} \Delta \lambda_{n} + \sum_{n_{A}=1}^{N} a_{\lambda_{n}}^{*} \Delta_{\lambda_{n}\lambda_{n_{A}}} c_{A} \right) \\ \times \int_{0}^{1} \left( I_{\lambda_{n}}^{+}(z, v) + I_{\lambda_{n}}^{-}(z, v) \right) v \, \mathrm{d}v; \tag{9}$$

$$\begin{split} \psi &= 2\pi (\rho^{\mathrm{atm}})^{-1} \sum_{n=1}^{\mathrm{S}} \sum_{n_{A}}^{N} \left\{ J_{\lambda_{n_{A}}}^{\mathrm{atm}} \left( \Delta_{\lambda_{n}\lambda_{n_{A}}} / \Delta \lambda_{n_{A}}^{\mathrm{ef}} \right) \right. \\ & \times \int_{0}^{1} \left( I_{\lambda_{n}}^{+}(z,v) + I_{\lambda_{n}}^{-}(z,v) \right) \mathrm{d}v \right\}. \end{split}$$
(10)

As factors of laser radiation effect are determined by expressions (8)–(10), the heat and mass transfer and stress state in considered semitransparent layer with admixture are described by the set of model equations [5,6] consisting of thermal conductivity equation, admixture diffusion equation, admixture energy balance equation as well equations of quasistationary thermoelasticity completed by correspondent initial and boundary equations. These equations for one-dimensional infinite semitransparent layer with admixture are [6]

$$\frac{\partial^2 T}{\partial z^2} - \frac{\partial T}{\partial \tau_T} + \frac{\hbar^2}{\kappa} \rho_M Q = 0, \tag{11}$$

$$\frac{\partial}{\partial z} \left( \frac{D_A(E_A)}{D_{0A}} \frac{\partial c_A}{\partial z} \right) - \frac{\partial c_A}{\partial \tau_c} = -\frac{\partial}{\partial z} \left( \frac{M_A}{D_{0A}} \frac{D_A(E_A)c_A}{E_A} \frac{q_A}{T} \frac{\partial T}{\partial z} \right),\tag{12}$$

$$D_A = D_{0A} \exp(-E_{\rm act}/E_A),$$
 (13)

$$E_A = \frac{2}{3} \frac{M_A h^2}{D_{0A}} \psi_A \tau_c + RT_0 \leqslant E_A^{\max}, \quad \text{if } L_A(z) \ge 1, \qquad (14a)$$

$$E_A = RT, \quad \text{if } L_A(z) < 1, \tag{14b}$$

$$L_{A}(z) = \frac{N_{\rm pt}(z)d^{2}}{p(z)}, \quad N_{\rm pt}(z) = \sum_{n_{A}}^{N} I_{\lambda_{n_{A}}}(z) \Delta \lambda_{n_{A}}^{\rm ef} / (\hbar \bar{c} / \lambda_{n_{A}}),$$
(15)

$$\frac{\partial^2 \sigma_{xz}}{\partial z^2} + h \frac{\partial F_x}{\partial z} = 0, \quad \frac{\partial^2 \sigma_{yz}}{\partial z^2} + h \frac{\partial F_y}{\partial z} = 0, \quad \frac{\partial^2 \sigma_{zz}}{\partial z^2} + h \frac{\partial F_z}{\partial z} = 0,$$
(16)

$$\frac{\partial^2 \sigma_{xx}}{\partial z^2} = \frac{1}{1 - \bar{v}} \left( \overline{E} \frac{\partial^2 L}{\partial z^2} + \bar{v}h \frac{\partial F_z}{\partial z} \right),$$
$$\frac{\partial^2 \sigma_{yy}}{\partial z^2} = \frac{1}{1 - \bar{v}} \left( \overline{E} \frac{\partial^2 L}{\partial z^2} + \bar{v}h \frac{\partial F_z}{\partial z} \right); \tag{17}$$

$$L(T, c_A) = \alpha_T (T - T_0) + w(c_A - c_0).$$
(18)

Here  $\tau_T = k_T t/h^2$ ;  $\tau_c = D_{0A}t/h^2$ . Neglecting effect of ponderomotive forces  $F_x, F_y, F_z$  on stress state of the layer, from Eqs. (16) we obtain that stress tensors components  $\sigma_{xz} = \sigma_{yz} = \sigma_{zz}$  become zero, that is, plane stress state is realized in the layer so that Eqs. (16) may be omitted while Eqs. (17) are simplified to appear as

$$\frac{\partial^2 \sigma_{xx}}{\partial z^2} = \frac{1}{1 - \bar{v}} \overline{E} \frac{\partial^2 L}{\partial z^2}, \quad \frac{\partial^2 \sigma_{yy}}{\partial z^2} = \frac{1}{1 - \bar{v}} \overline{E} \frac{\partial^2 L}{\partial z^2}.$$
 (19)

In Eqs. (12) and (13)  $E_A$  denotes admixture molar energy. Dependent on introduced in [6] parameter  $L_A$ , accounting for correlation between radiation source parameters and diffusive characteristics of admixture, values of quantity  $E_A$  may be equal to or exceed the molar energy of thermal vibration of matrix, RT. Physically, the equality  $E_A = RT$  means that radiation produces no specific effect on admixture. Meantime, the inequality  $E_A > RT$  corresponds to the case when admixture unit mass absorbs more radiation energy than matrix unit mass because of admixture absorption coefficient being higher than that of matrix, thus increasing diffusion rate due to appropriate rise of admixture diffusivity  $D_A$  [6] (see Eq. (13)). Procedure for determination of maximum value  $E_A^{\text{max}}$  of energy  $E_A$  is considered in detail in [6].

Eqs. (11)–(15) and (18) and (19) form the set of model equations for determination temperature and admixture concentration distributions in the layer as well as its stress state. Consider now correspondent boundary conditions.

Since layer's surroundings  $z_* < 0$  are vacuumed, while domain  $z_* > h$  represents real gaseous medium of a given admixture concentration  $c_{sur}$  and temperature  $T_{sur}$ , the thermal boundary conditions appear as

$$\frac{\partial T(0,\tau_T)}{\partial z} = 0, \quad \frac{\partial T(1,\tau_T)}{\partial z} + Bi_T(T-T_{\rm sur}) = 0, \tag{20}$$

where  $Bi_T = \bar{\alpha}h/\kappa$  is appropriate Biot criterium.

Let us derive mass exchange boundary conditions. Mass exchange rate at layer's surface may be presented as follows [10]

$$\theta_A = -\gamma^A m_{\rm sur} + m_s / \tau^A. \tag{21}$$

Here  $\gamma^4 m_{\text{sur}}$  and  $m_s/\tau^4$  are masses of admixture respectively absorbed at unit surface and desorbed from it, in unit time. If monolayer's thickness  $h_m$  is known, densities of admixture on layer's boundary and in surroundings may be defined using quantities  $m_s$  and  $m_{\text{sur}}$ . Then condition (21) in terms of dimensionless admixture concentration  $c_A$  appear as

$$\theta_A = -\gamma^A h_m \rho_A^{\text{sur}} + h_m \rho_A / \tau^A = \beta^A (-c_A - \kappa^A c_A^{\text{sur}}), \qquad (22)$$

where  $\beta^A = \rho_M h_m / \tau^A$ ,  $\kappa^A = \tau^A \gamma^A$  are mass exchange coefficients,  $c_A^{\text{sur}} = \rho_A^{\text{sur}} / \rho_A$  is admixture concentration in surroundings. Using above correlations mass exchange boundary conditions are written in the form

$$\begin{aligned} \mathbf{J}_{A_{z}}(0,\tau_{c}) &= Bi_{c}c_{A}(0,\tau_{c}), \\ &- \mathbf{J}_{A_{z}}(1,\tau_{c}) = Bi_{c}[c_{A}(1,\tau_{c}) - \kappa^{4}c_{A}^{\mathrm{sur}}], \end{aligned}$$
(23)

where  $Bi_c = \beta^A h / \rho_M D_{0A}$  is Biot criterium for mass exchange.

At edges  $x \to \infty$ ;  $y \to \infty$  of the layer following mechanical fixing conditions are accepted

$$\int_0^1 \sigma_{xx} \, \mathrm{d}z = 0; \int_0^1 \sigma_{yy} \, \mathrm{d}z = 0.$$
 (24)

### 4. Solution procedure

Solution procedure is grounded on iterative technique. In doing so, (p + 1)th approximation of solution of Eq. (7) is

$$I_{\lambda_n(p+1)}^+(0,v) = I_{\lambda_n}^0 \left[ 2R_{\lambda_n} E_3(\theta_{\lambda_n(p)}^u) A_{\lambda_n(p)} + \begin{cases} \delta(\xi - \xi_0) & \text{for } v \leqslant v_* \\ 0 & \text{for } v > v_* \end{cases} \right],$$
(25)

$$I^{-}_{\lambda_{n}(p+1)}(1,v) = I^{0}_{\lambda_{n}}A_{\lambda_{n}(p)}, \quad I^{0}_{\lambda_{n}} = k(n_{\lambda_{n}})^{2}(1-R_{\lambda_{n}})I^{S}_{\lambda_{n}},$$

where

$$\begin{split} A_{\lambda_{n}(p)} &= \frac{2R_{\lambda_{n}}v_{0}\exp[-\theta_{\lambda_{n}(p)}^{u}/v_{0}]}{1 - \left[2R_{\lambda_{n}}E_{3}(\theta_{\lambda_{n}(p)}^{u})\right]^{2}},\\ v_{0} &= \sqrt{1 - \left(1/n_{\lambda_{n}}\cdot\sin\xi_{0}\right)^{2}},\\ v_{0} &= \sqrt{1 - \left(1/n_{\lambda_{n}}\cdot\sin\xi_{0}\right)^{2}}; \quad \theta_{\lambda_{n}(p)}^{u} = \int_{0}^{1}\theta_{\lambda_{n}}(c_{A(p)})\,\mathrm{d}z. \end{split}$$

Factors of laser radiation effect in (p + 1)th iteration consequently are

$$\rho_{M}Q_{(p+1)} = 2\pi \sum_{n=1}^{S} I_{\lambda_{n}}^{0} \Gamma_{\lambda_{n}(p)}(z) \left( a_{\lambda_{n}}^{M} \Delta \lambda_{n} + \sum_{n_{A}=1}^{N} a_{\lambda_{n}}^{*} \Delta_{\lambda_{n}\lambda_{n_{A}}} c_{A(p)} \right);$$
(26)
$$F_{z}^{(p+1)} = 2\pi \bar{c}^{-1} \sum_{n=1}^{S} I_{\lambda_{n}}^{0} \Pi_{\lambda_{n}(p)}(z) \left( a_{\lambda_{n}}^{M} \Delta \lambda_{n} + \sum_{n_{A}=1}^{N} a_{\lambda_{n}}^{*} \Delta_{\lambda_{n}\lambda_{n_{A}}} c_{A(p)} \right);$$
(27)
$$\psi_{A(p+1)} = 2\pi (\rho^{\text{atm}})^{-1} \sum_{n=1}^{S} \sum_{n=1}^{N} I_{\lambda_{n}}^{0} J_{\lambda_{n}}^{\text{atm}} \left( \Delta_{\lambda_{n}\lambda_{n}} / \Delta \lambda_{\lambda_{n}}^{\text{ef}} \right) \Gamma_{\lambda_{n}(p)}(z).$$

$$\psi_{A(p+1)} = 2\pi (\rho^{\text{atm}})^{-1} \sum_{n=1} \sum_{n_A=1} I^0_{\lambda_n} J^{\text{atm}}_{\lambda_{n_A}} \Big( \Delta_{\lambda_n \lambda_{n_A}} / \Delta \lambda^{\text{ef}}_{\lambda_{n_A}} \Big) \Gamma_{\lambda_n(p)}(z).$$
(28)

Here

$$\begin{split} \Gamma_{\lambda_n(p)}(z) &= \{ 2R_{\lambda_n} E_3(\theta^u_{\lambda_n(p)}) E_2(\varphi_{\lambda_n(p)}(z)) \\ &+ E_2(\zeta_{\lambda_n(p)}(z)) \} A_{\lambda_n(p)} + \exp(-\varphi_{\lambda_n(p)}(z)/v_0); \end{split}$$

$$\begin{split} \Pi_{\lambda_n(p)}(z) &= \{ 2\kappa_{\lambda_n} E_3(\theta_{\lambda_n(p)}) E_3(\varphi_{\lambda_n(p)}(z)) \\ &- E_3(\zeta_{\lambda_n(p)}(z)) \} A_{\lambda_n(p)} + v_0 \exp(-\varphi_{\lambda_n(p)}(z)/v_0); \end{split}$$

$$egin{aligned} arphi_{\lambda_n(p)}(z) &= \int_0^z heta_{\lambda_n(p)} c_{A(p)}(z^*) \,\mathrm{d} z^*, \ \zeta_{\lambda_n(p)}(z) &= -\int_1^z heta_{\lambda_n(p)} c_{A(p)}(z^*) \,\mathrm{d} z^*, \end{aligned}$$

where  $E_n(x)$  is integroexponential function [9]. As zero approximation of admixture concentration the initial admixture concentration is taken.

### 5. Numerical results

Numerical investigations were carried out for the layer made of glass S93-2 containing water vapor or carbon monoxide admixture, as the most frequently met impurities in such type glass [10], initially homo-

Table 1			
Parameters of ra	diative bands	of different lasers	

Laser	Center of the band, µm	Band's width, μm
Ar-He (L1)	4.28	6.02
Xe-He (L2)	7.47	10.89
Cs (L3)	5.19	3.98
N <sub>2</sub> -He (L4)	2.85	3.61

geneously distributed with concentration 0.001. Initial uniform temperature of the layer equals  $T_0 = 300$  K. Thermal, mechanical and radiative characteristics of the layer were taken from [8], while radiative characteristics of gaseous admixture were borrowed from [11]. Spectral absorption coefficient of the matrix was approximated by piece-wise homogeneous function [6]

$$a_{\lambda}^{M} = \begin{cases} a_{1}; & \text{if } 0 \leq \lambda < \lambda_{\text{cut}}; \\ a_{2}; & \text{if } \lambda_{\text{cut}} \leq \lambda < \infty \end{cases}$$

where  $\lambda_{cut}$  is cutoff wavelength. Laser radiation produced by real lasers was considered. In Table 1 laser characteristics [8] are presented.

In the case of each laser the net radiation flux

$$I^{\rm S} = \int_0^\infty I^{\rm i}_{\lambda_n} \,\mathrm{d}\lambda$$

entering the layer was taken the same and equal to that produced by Ar–He laser (L1), owing to appropriate choice of factor k in Eq. (1), namely k = 1; 0.553; 1.513; 1.668 for lasers L1, L2, L3, L4, respectively. In all calculations normal incidence of laser beam on surface z = 0 of the layer was considered ( $\xi_0 = 0$ ).

Numerical results suggest that redistributions of H<sub>2</sub>O or CO admixtures due to laser irradiation during not more than 2 h do not cause essential change of heat sources in the layer, in comparison to those calculated at initial admixture distribution. Dimensionless heat sources distributions ( $Q_* = Q/Q(z=0)$ ) in 1 cm thick layer subjected to moderate laser radiation from lasers L1-L4 are presented in Fig. 2 by curves 1-4 respectively. It should be noted that these distributions are, in fact, insensitive to whether H<sub>2</sub>O or CO admixture is present in the layer. Heat sources distributions in layers of various thickness subjected to lasers L1 and L2 are depicted by curves 1-3 in Figs. 3 and 4 respectively. One can see from Figs. 3 and 4 that as layer's thickness increases the heat sources tend to be located closer to the surface. It could be concluded from analysis of Fig. 2 that heat sources distributions are caused mainly by the correlation between locations of laser radiation band and cutoff wavelength. To describe that correlation quantitatively dimensionless parameter P is introduced equal to the correlation between the amount of laser radiation energy falling into spectral range between cutoff



Fig. 2. Distributions of dimensionless heat sources in 1 cm thick layer subjected to radiation from different lasers. L1 (1); L2 (2); L3 (3); L4 (4).



Fig. 3. Distributions of dimensionless heat sources in the layers of various thickness subjected to radiation from L1 laser. h = 1 cm (1); 3 cm (2); 5 cm (3).

wavelength and right-hand side of laser band and that between cutoff wavelength and left-hand side of laser band. Meanings of parameter P are 0.32; 0.07; 0; and 1.84 for lasers L1–L4 correspondently and, as it might be seen from Fig. 2, as value of this parameter decreases the heat sources distributions become more nonuniform. Meantime, if values of parameter P are close, as it is for L2 and L3, heat sources distributions dependence upon laser band location in the spectrum is negligible (see curves 2 and 3 in Fig. 2).

Time evolutions of the temperature in cross-section z = 0.5 of 1 cm thick layer containing CO subjected to various laser radiation (L1–L4) are presented in Fig. 5.



Fig. 4. Distributions of dimensionless heat sources in the layers of various thickness subjected to radiation from L2 laser. h = 1 cm (1); 3 cm (2); 5 cm (3).



Fig. 5. Time evolutions of the temperature caused by different lasers at z = 0, 5 of 1 cm thick layer. L1 (1); L2 (2); L3 (3); L4 (4).

Here and further on, laser beam spectral intensity for L1 was chosen equal 500 W/(m<sup>2</sup> ster  $\mu$ m) in order to guarantee the layer's temperature not exceeding glass transformation level [12] during heating interval (2 h). Temperature distributions over layer's thickness after 2 h of laser irradiation are presented in Fig. 6. It might be seen from the figure that the smaller parameter *P* is the more nonuniform temperature distribution in the layer is observed. Figs. 7 and 8 illustrate temperature distributions in layers of various thickness after 2 h of irradiation produced by lasers L1 and L4 respectively. It could be observed from the figures that the larger layer's thickness is the more nonuniform temperature profiles appear.



Fig. 6. Temperature distributions in 1 cm thick layer subjected to radiation from different lasers. L1 (1); L2 (2); L3 (3); L4 (4).



Fig. 7. Temperature distributions in the layers of various thickness subjected to radiation from L1 laser. h = 1 cm (1); 3 cm (2); 5 cm (3).



Fig. 8. Temperature distributions in the layers of various thickness subjected to radiation from L1 laser. h = 1 cm (1); 3 cm (2); 5 cm (3).



Fig. 9.  $H_2O$  distribution in 1 cm thick layer subjected to radiation from different lasers. L1, L2, L3 (1); L4 (2).



Fig. 10. CO distribution in 1 cm thick layer subjected to radiation from different lasers. L1, L2, L3 (1); L4 (2).

Admixture concentration distributions in 1 cm thick layer containing H<sub>2</sub>O (Fig. 9) or CO (Fig. 10) after 2 h of laser irradiation are presented by curves 1 and 2. It is easily seen from the figures that lasers L1-L3 cause the same diffusion rate of each considered admixtures while laser L4 does not induce diffusion at all. It might be explained by the fact that radiation band of laser L4 (1.04-3.65 µm) intersects just a single weak absorption band of CO (band's center 2.35 µm) and one of three principal absorption bands (band's center 2.70 µm) of H<sub>2</sub>O. Therefore, consideration of correlations between laser radiation bands and admixture absorption bands is important at selection of certain laser with the aim of increasing of admixture diffusion rate. It should be also stated that contributions of thermodiffusion flux and diffusion flux caused by ponderomotive forces (at considered spectral radiation intensities  $I_{i}^{S}$  not exceeding 10<sup>4</sup>  $W/(m^2 \text{ ster } \mu m))$  into net diffusion flux are negligible as it is also held for the case of thermal radiation [5,6].

Normal stresses ( $\sigma = \sigma_{xx} = \sigma_{yy}$ ) distributions in the CO-contained 1 cm thick layer rigidly fixed at the edges  $x, y \to \pm \infty$  (condition (15)) after 2 h of irradiation produced by lasers L1–L4 (curves 1–4) are presented in Fig. 11. Figs. 12 and 13 illustrate normal stresses



Fig. 11. Normal stresses distributions in 1 cm thick layer subjected to radiation from different lasers. L1 (1), L2 (2), L3 (3); L4 (4).



Fig. 12. Normal stresses distributions in the layers of various thickness subjected to radiation from L1 laser. h = 1 cm (1); 3 cm (2); 5 cm (3).



Fig. 13. Normal stresses distributions in the layers of various thickness subjected to radiation from L4 laser. h = 1 cm (1); 3 cm (2); 5 cm (3).

distributions in layers or various thickness (curve 1–3) subjected to lasers L1 and L4, respectively. It is seen from Figs. 11–13 that maximum compressive and tensile stresses increase as values of parameter P decrease. Figs. 12 and 13 also demonstrate that the increase of layer's thickness leads to such a rise of maximum tensile stresses so that their values might even exceed permissible level (see curves 2, 3 in Fig. 12 and curve 3 in Fig. 13) for the considered glass layer.

### 6. Conclusions

We have investigated the effect of moderate laser radiation upon heat and mass transfer in two-component elastic semitransparent layer consisting of elastic matrix and gaseous admixture. Numerical investigations suggest that different infrared lasers cause different mass and temperature distributions in the layer what should be attributed to the distinct effect of the correlations between spectral radiative characteristics of different lasers and absorptive characteristics of matrix and admixture. In particular, expected strong correlation between effect of different lasers and diffusion rate of certain admixtures might be used in selection of suitable laser for realization of improved technologies of laser-assisted drying or degassing of semitransparent materials. It is also established that contributions of thermodiffusion flux and diffusion flux due to ponderomotive forces, caused by moderate laser radiation, into net diffusion flux are negligible as it is also held for the case of thermal radiation.

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